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# ANALYSIS OF INTENSIVE EVAPORATION AND CONDENSATION

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**Abstract**—Applying the principles of the molecular-kinetic theory, the basic relationships are studied which govern the processes of intensive evaporation and condensation. One-dimensional steady-state problems are considered. Through application of an approximate method, developed by the authors, a closed description of the processes of intensive evaporation and condensation is obtained. The calculation data are presented which characterize the basic regularities in the above processes. These are compared with the experimental data of Necmi and Rose [23] on intensive condensation of mercury. Based on the results obtained, simple interpolation formulae are suggested. The findings of the analysis check well with the known numerical solutions of the problems of intensive evaporation and condensation. In the region of low-intensity processes, the proposed formulae go over into the known linear theory relations.

## NOMENCLATURE

| | |
|---|---|
| $C$, | $= \xi - \mathbf{u}$, intrinsic velocity of molecules; |
| $C_p$, | vapour specific heat at constant pressure; |
| $f$, | distribution function; |
| $f_0$, | local Maxwellian distribution function; |
| $\dot{H}_x$, | Boltzmann $H$-function flux along coordinate $x$; |
| $j$, | specific mass flux; |
| $j_p$, | specific momentum flux; |
| $j_e$, | specific energy flux; |
| $k$, | Boltzmann constant; |
| $K$, | specific heat ratio of vapour; |
| $l$, | mean free path of vapour molecules; |
| $M$, | $= \dfrac{u}{(KRT)^{1/2}}$, Mach number; |
| $m$, | mass of molecule; |
| $n$, | number density of vapour molecules; |
| $n_0$, | number density at $x = 0$; |
| $n_s$, | equilibrium number density corresponding to temperature $T_s$; |
| $n_\infty$, | number density far from the interphase surface; |
| $P$, | pressure; |
| $P_s$, | saturation vapour pressure corresponding to temperature $T_s$; |
| $P_\infty$, | vapour pressure far from the interphase surface; |
| $P_{ij}$, | stress tensor, $p_{ij} = P_{ij} - P\delta_{ij}$; |
| $Pr$, | vapour Prandtl number; |
| $q$, | specific heat flux; |
| $R$, | vapour gas constant; |
| $r$, | specific heat of evaporation; |
| $T$, | temperature; |
| $T_s$, | interface temperature; |
| $T_\infty$, | temperature far from the interphase surface; |
| $u$, | macroscopic vapour flow velocity; |
| $x,y,z$, | coordinates. |

Greek symbols

| | |
|---|---|
| $\beta$, | coefficient of evaporation (condensation); |
| $\lambda$, | thermal conductivity of vapour; |
| $\mu$, | vapour viscosity; |
| $\xi$, | velocity of a molecule in a laboratory coordinate system $x,y,z$; |
| $\xi_x,\xi_y,\xi_z$, | components of velocity $\xi$ in the coordinate system $x,y,z$; |
| $\rho$, | $= m \cdot n$, density; |
| $\rho_s$, | equilibrium density corresponding to temperature $T_s$; |
| $\rho_\infty$, | vapour density far from interphase surface. |

## 1. INTRODUCTION

THE PAPER is concerned with the study of laws governing intensive processes of evaporation and condensation. Briefly the problem under investigation consists in the following. An analysis is performed of a one-dimensional steady-state process of evaporation (condensation) of liquid or a solid body surrounded by its own vapour (Fig. 1). The interphase surface temperature, $T_s$, and thus the equilibrium saturation pressure, $P_s$, which corresponds to this temperature, are assumed to be known. It is required to determine mass, momentum and energy fluxes at some given vapour properties far from the interface ($T_\infty, P_\infty, \ldots$).

An essential point of the mechanism of evaporation and condensation is a unique character of the non-equilibrium state of vapour near the interface. The surface layer of vapour is formed of two opposite flows of molecules with different distribution functions (spectra). The flow of molecules from the surface is determined by the surface emission of molecules. In accordance with the common approach [1], we shall assume that in the problems considered the diffusion scheme of evaporation is realized, i.e., the distribution function of molecules moving from the interface has the form

$$f_s = \frac{n}{(2\pi R T_s)^{3/2}} \cdot e^{-\xi^2/2RT_s}. \qquad (1.1)$$

The density of molecules, $n$, in equation (1.1) is equal to, or less than, the saturated vapour density, $n_s$, at the surface temperature

$$n = \beta n_s. \qquad (1.2)$$

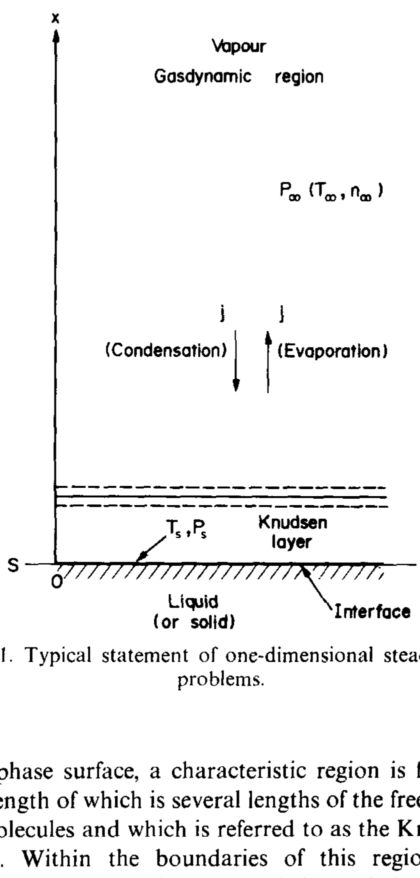

FIG. 1. Typical statement of one-dimensional steady-state problems.

The coefficient $\beta \leqslant 1$, called the evaporation (condensation) coefficient, is an averaged characteristic of the surface conditions and of the physical nature of a condensed phase. The opposite flow of molecules is formed in the vapour layers farther removed from the wall and its spectrum is different. Near the interphase surface, a characteristic region is formed the length of which is several lengths of the free paths of molecules and which is referred to as the Knudsen layer. Within the boundaries of this region, the ordinary macroscopic characteristics and the resulting gradient-type transfer equations (the Fourier law, Newton's law of viscous friction) turn to be invalid. Thus, the whole vapour space may be divided into two regions (Fig. 1): a Knudsen layer and an outer region (the region of a gasdynamic flow).* A fairly detailed analysis of vapour behaviour in the Knudsen layer from the viewpoint of the molecular-kinetic theory (solution of the Boltzmann kinetic equation or of its models) has been carried out for the region of low-intensity processes ($M \ll 1$). Using the moment method, the authors of [3, 4] received the following expressions for the extrapolated temperature and pressure jumps at the interface as functions of $j$ and $q$

$$\frac{T_s - T(0)}{T_s} = 0.454\hat{j} + 2.686\hat{q}_1,$$
$$\frac{P_s - P(0)}{P_s} = \frac{1 - 0.399\beta}{\beta} \cdot 2(\pi)^{1/2} \cdot \hat{j} + 1.107\hat{q}_1, \qquad (1.3)$$

where $\hat{j} = j/(\rho_s C_s)$ is the dimensionless mass flux, $\hat{q}_1 = 2/5 \cdot q_1/(P_s C_s)$ the dimensionless heat flux by heat conduction, $C_s = (2RT_s)^{1/2}$.

In a number of later works, [5–7], these relations were confirmed by applying some other methods for solution of kinetic equations. On the basis of the results obtained in [3, 4] the conclusions were drawn about the vapour state near the interface. Thus, in accordance with [3], at $\beta = 1$ the vapour becomes super-saturated during evaporation and superheated during condensation.

With increase in the rate of evaporation or condensation ($M \sim 1$), the departure of vapour from the equilibrium state in the Knudsen layer also increases. This appreciably complicates description of these processes. Throughout the years 1930–1960 attempts were undertaken by a number of researchers [8–10] to develop a correlation for the evaporation intensity $j$ by refining the well-known simple Herts–Knudsen formula (taking into account the vapour flow motion, etc.). A most comprehensive review of these efforts is given in [9]. It might be well to point at inconsistency of the calculation schemes suggested by the authors of [8–10]. Usually, the description does not allow for the actual physical situation and vapour state near the surface. This has objectively called for going over to a rigorous formulation of the basic kinetic equations. The first accurate investigations in the finite-intensity region were undertaken by the authors of [11] who, by the use of the integral method, obtained numerical solution of the BGK model kinetic equation. The analysis performed by these authors has revealed that with considerable increase in the rate of evaporation and condensation, the temperature and density jumps become commensurate with the magnitudes of temperature and density of the vapour. It has been specially noted that "steady 'in vacuo' vaporization is impossible in the process of evaporation" and that the limiting mass flow rate of emitted molecules, calculated from formula (1.1), is not attainable, i.e., there is a maximum vapour flow rate which is less than $P_s/(2\pi R T_s)^{1/2}$. Unfortunately, reduced accuracy of calculations in the region of large mass fluxes has not allowed these authors to obtain the value of the maximum mass flux.

The first, approximate, calculation of the maximum flow rate in the course of evaporation appears to be carried out in [12] where, on the basis of the BGK model equation, an unsteady problem of vapour expansion on its vaporization into vacuum was solved numerically. It was shown that after a

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* For details, see [2].

time lapse of about 20 "free path time" intervals, stabilization of the return molecule flow is observed and the mass flux decreases from its initial value $P_s/(2\pi RT_s)^{1/2}$ by about 20%.

Thus, in the steady-state evaporation regime with a maximum mass flux, approximately 1/5 of the number of emitted molecules returns to the surface and condenses. According to the calculations performed in this work, the temperature of the vapour far from the surface in the maximum evaporation regime (maximum mass flux) is $T_\infty = 0.69T_s$. The parameters obtained here for the regime of the maximum evaporation rate agree well with the approximate estimates of this very author but reported earlier in [13, 14].

In [14] it was also noted that in the regime of maximum evaporation rate the state of vapour is distinguished by considerable supersaturation. In a recently published work [15], the process of intensive evaporation was treated by the Monte-Carlo method with the use of the scheme suggested by Bird [16]. In the region of low and moderate intensities the results of [15] are in a good agreement with the data of [11] and, for the limiting evaporation regime, the process characteristics are consistent with the conclusions drawn in [13, 14].

The region of intensive condensation has not been so thoroughly investigated as that of evaporation. Reference [11] appears to be the only one to obtain discreet calculations on some condensation regimes. It also contains the conclusion concerning asymmetry in the description of intensive evaporation and condensation which does not manifest itself in the region of validity of the linear theory. An example of intensive condensation is considered in [17]. The analysis was carried out only for two Mach numbers of the incident flow: $M_\infty = 0.855$ and $M_\infty = 1.5$. All information reported in the works quoted has been obtained either on the basis of the BGK model equation or by computer modelling of the very process of molecular collisions with the use of the Monte-Carlo method. Solution of the above problems on the basis of the general Boltzmann equation has not been obtained as yet.

In concluding this brief survey, we wish to emphasize again that the region of the processes of intensive evaporation and condensation has been little studied to date. Consequently, particularly urgent becomes investigation of approximate methods of solution of the problems outlined.

One of the possible approximate methods for the analysis of intensive evaporation and condensation is given in the present work. Section II of the paper deals with intensive evaporation,* while Section III, with intensive condensation.

## II. INTENSIVE EVAPORATION

In conformity with the general statement of the problem under consideration, we assume for the solution of the problem of intensive evaporation that $T_s$ and $n_s$ are known as well as one of the parameters of vapour ($P_\infty$, $T_\infty$, $n_\infty$) at a distance from the interface. As a rule, such an external parameter, preassigned for the system, is the pressure of the surrounding medium, $P_\infty$. It is clear that knowledge of this quantity is sufficient for obtaining the solution. Since the whole space occupied by vapour may be subdivided into two regions (see above), we, first, shall study the behaviour of vapour in the gasdynamic region on the basis of continuum mechanics considerations and then investigate the processes occuring in the Knudsen layer on the basis of the kinetic theory. The analysis of this and of all the subsequent problems considered here will be made in the rectangular coordinate system in which the plane $y,z$ coincides with the interface (Fig. 1).

### (A) *Gasdynamic flow region*

For the considered problem the continuity equations of motion and energy of the vapour flow have the form

$$\rho u = \rho_\infty u_\infty, \tag{2.1}$$

$$P+\rho u^2 - 4/3\mu\frac{\mathrm{d}u}{\mathrm{d}x} = (P+\rho u^2)_\infty, \tag{2.2}$$

$$\rho u\left(C_pT+\frac{u^2}{2}\right)-\lambda\frac{\mathrm{d}T}{\mathrm{d}x}-\tfrac{4}{3}\mu u\frac{\mathrm{d}u}{\mathrm{d}x} = \rho_\infty u_\infty\left(C_pT+\frac{u^2}{2}\right)_\infty. \tag{2.3}$$

The subscript ∞ refers to the equilibrium Euler region of the vapour flow which is eventually attained by vapour. The LHS of expressions (2.1)–(2.3) are written for an arbitrary point in the gasdynamic region. For $Pr = 3/4$, which is valid for the majority of gases and vapours, equation (2.3) yields

$$j\bar{h}-\tfrac{4}{3}\cdot\mu\cdot\frac{\mathrm{d}\bar{h}}{\mathrm{d}x} = j\bar{h}_\infty, \tag{2.4}$$

where $j = \rho u$, $\bar{h} = C_pT+(u^2/2)$. Denoting

$$Z = \tfrac{3}{4}|j|\int_0^x\frac{\mathrm{d}x}{\mu}$$

and taking into consideration that for evaporation $j > 0$, equation (2.4) may be reduced to

$$\bar{h}-\frac{\mathrm{d}\bar{h}}{\mathrm{d}Z} = \bar{h}_\infty. \tag{2.5}$$

The only solution of the above equation satisfying the condition $\mathrm{d}\bar{h}/\mathrm{d}Z|_{Z\to\infty}\to 0$ (transition to the Euler equilibrium region) is

$$\bar{h} = \bar{h}_\infty. \tag{2.6}$$

The equation of motion (2.2), after some manipulations, is reduced to

$$\frac{K-1}{K}\cdot H_\infty\left(\frac{\bar{h}}{\bar{h}_\infty}-V\right)+\frac{K+1}{K}(V^2-V)\mp\frac{\mathrm{d}V^2}{\mathrm{d}Z} = 0, \tag{2.7}$$

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*Statement of the intensive evaporation problem was given in a very brief form in [18].

where

$$V = \frac{u}{u_\infty},\ H_\infty = \frac{\bar{h}_\infty}{u_\infty^2/2},\ K = \frac{C_p}{C_v},$$

(the sign "−" is used for evaporation and the sign "+", for condensation).

According to (2.6), $\bar{h}/\bar{h}_\infty = 1$. This allows a conclusion that

$$V = 1 \tag{2.8}$$

is the solution of equation (2.7) satisfying the conditions

$$(\mathrm{d}V/\mathrm{d}Z)_{Z\to\infty} \to 0,\quad V|_{Z\to\infty} = 1.$$

(B) *Knudsen layer*

The Knudsen layer is made up of the two opposite flows of molecules with different distribution functions (see Section I). The flow of molecules from the surface is governed by the surface emission of molecules and is described by the Maxwellian distribution function (1.1). The incident flow of molecules is formed in the hydrodynamic region and it "remembers" the state and motion of vapour in this region. The Euler equilibrium flow is described by the following distribution function

$$f_\infty = n_\infty\left(\frac{m}{2\pi kT_\infty}\right)^{3/2}\cdot\mathrm{e}^{-\frac{m(\xi-\mathbf{u}_\infty)^2}{2kT_\infty}}$$

$$\begin{aligned}\infty > \xi_y &> -\infty\\ \infty > \xi_z &> -\infty\\ \infty > \xi_z &> -\infty.\end{aligned} \tag{2.9}$$

Obviously, the form of this function at intermediate points will be more complex. An accurate solution of the problem consists in finding such a distribution function which would satisfy the Boltzmann kinetic equation at every point in the Knudsen layer and the following boundary conditions

$$\begin{aligned} x = 0,\quad f &= f_s \quad \text{for } \xi_x > 0,\\ x \to \infty,\quad f &= f_\infty \quad \text{for all } \xi_x. \end{aligned}$$

In order to obtain a solution of the Boltzmann equation by the approximate moment method, one needs an approximation on the distribution function. We have considered some possible structures of the distribution function approximations suitable for description of the process at every point of the Knudsen layer. This study allowed us to establish an interesting feature peculiar to a particular kind of discontinuous four-moment approximations.* It is this: in order to determine the vapour parameters in the region of the gasdynamic flow, it is sufficient that only the system of conservation equations be solved. And really, with a four-moment approximation of the distribution function available, it is possible to write the expressions for the mass, momentum and energy fluxes in the zones $x = 0$ and $x \to \infty$ and then, by setting these expressions equal, to obtain three conservation equations which comprise seven unknown quantities: three at $x \to \infty$ [the Maxwellian distribution function (2.9)] and four at $x = 0$ (by virtue of the four-moment approximation). The diffusion nature of evaporation (see above) implies three boundary conditions for the moments which describe the state of molecules in the region $0 < \xi_x < \infty$. Thus, there are six equations (3 conservation equations and 3 boundary conditions at $x = 0$) for determining seven unknown values. In accordance with the statement of the problem, one parameter for $x \to \infty$ is to be pre-assigned. This closes the system and all the unknown values are unambiguously determined.

When defining the structue of the required four-moment approximation one must bear in mind the following:

(1) in accordance with (2.6) and (2.8), at the outer boundary of the Knudsen layer the Euler flow is realized which is described by the distribution function (2.9);

(2) with small deviation of the vapour properties from equilibrium the results of calculation with the use of the four-moment approximation must go over into corresponding expressions of the linear theory.

The analysis of different four-moment approximations made it possible to identify the following function as best complying with the above requirements

$$f = \begin{cases} f_- = n_2(x)\left(\dfrac{m}{2\pi kT_\infty}\right)^{3/2}\cdot\mathrm{e}^{-\frac{m}{2kT_\infty}\cdot[(\xi_x-u_\infty)^2+\xi_y^2+\xi_z^2]} & \xi_x < 0\\ f_+ = n_1(x)\left[\dfrac{m}{2\pi kT_1(x)}\right]^{3/2}\cdot\mathrm{e}^{-\frac{m}{2kT_1(x)}\{[\xi_x-u_1(x)]^2+\xi_y^2+\xi_z^2\}}, & \xi_x > 0, \end{cases} \tag{2.10}$$

where $n_1(x)$, $n_2(x)$, $T_1(x)$, $u_1(x)$ are four moments varying along the coordinate $x$. With $x \to \infty$, approximation (2.10) transforms into the Maxwellian function (2.9). With $x = 0$, because of the diffusion nature of evaporation, approximation (2.10) takes on the form

$$f = \begin{cases} f_1 = Cn_\infty\left(\dfrac{m}{2\pi kT_\infty}\right)^{3/2}\cdot\mathrm{e}^{-\frac{m}{2kT_\infty}[(\xi_x-u_\infty)^2+\xi_y^2+\xi_z^2]} & \xi_x < 0\\ f_s = n_s\left(\dfrac{m}{2\pi kT_s}\right)^{3/2}\cdot\mathrm{e}^{-\frac{m\xi^2}{2kT_s}} & \xi_x > 0, \end{cases} \tag{2.11}$$

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*The velocity distribution functions considered are assumed to consist of two parts: $f_1$ (for $\xi_x > 0$) and $f_2$ (for $\zeta_x < 0$), $f_1$ containing three moments changing along $x$ and $f_2$, one moment changing along $x$.

where $Cn_\infty = n_2(0)$, $C$ is a dimensionless constant. To determine the vapour parameters in the gasdynamic flow region ($n_\infty$, $T_\infty$, $u_\infty$), we shall set up the mass, momentum and energy conservation equations for two sections: $S$–$S$ and that which is located in the Euler flow region, using respectively functions (2.11) and (2.9). First, let us introduce the dimensionless quantities

$$T'_\infty = \frac{T_\infty}{T_s}, \quad n'_\infty = \frac{n_\infty}{n_s}, \quad u'_\infty = \frac{u_\infty}{\left(\dfrac{kT_s}{2\pi m}\right)^{1/2}} \tag{2.12}$$

(hereafter the primes will be omitted).

With this notation, the dimensionless evaporation surface temperature $T_s = 1$ and the corresponding equilibrium number density along the saturation line $n_s = 1$, while the conservation equations take on the form

$$\begin{aligned} &1 - Cn_\infty\left(T_\infty^{1/2}\cdot\phi - \frac{u_\infty}{2}\psi\right) = n_\infty u_\infty, \\ &\tfrac{1}{2} + Cn_\infty\left[\left(\frac{T_\infty}{2} + \frac{u_\infty^2}{4\pi}\right)\psi - \frac{u_\infty T_\infty^{1/2}}{2\pi}\phi\right] = n_\infty T_\infty + \frac{n_\infty u_\infty^2}{2\pi}, \\ &\tfrac{1}{2} - \tfrac{1}{2}Cn_\infty\left[\left(T_\infty^{3/2} + \frac{T_\infty^{1/2}u_\infty^2}{8\pi}\right)\phi - \left(\tfrac{5}{8}T_\infty u_\infty + \frac{u_\infty^3}{16\pi}\right)\psi\right] = n_\infty u_\infty\left(\tfrac{5}{8}T_\infty + \frac{u_\infty^2}{16\pi}\right), \end{aligned} \tag{2.13}$$

where

$$\phi = \exp\left(-\frac{u_\infty^2}{4\pi T_\infty}\right), \quad \psi = 1 - \operatorname{erf}\left[\frac{u_\infty}{2(\pi T_\infty)^{1/2}}\right], \quad \operatorname{erf}\alpha = \frac{2}{(\pi)^{1/2}}\int_0^\alpha e^{-x^2}\,dx.$$

Solution of the above system of equations requires pre-assignment of one of the quantities $n_\infty$, $T_\infty$, $u_\infty$, $C$. Specification of $T_\infty$ seems to be most convenient because in this case equations (2.13) will successively yield $u_\infty$, $n_\infty$, $C$.

In this fashion the system of equations (2.13) has been calculated for $T_\infty$ varying from 0.998 to 0.660 with a step 0.002. From the known values of $n_\infty$, $T_\infty$, $u_\infty$ and $C$ the mass, momentum and energy fluxes as well as the Mach number, $M$, in the region of the Euler flow were determined. The calculation results are presented in Table 1 and in Fig. 2.

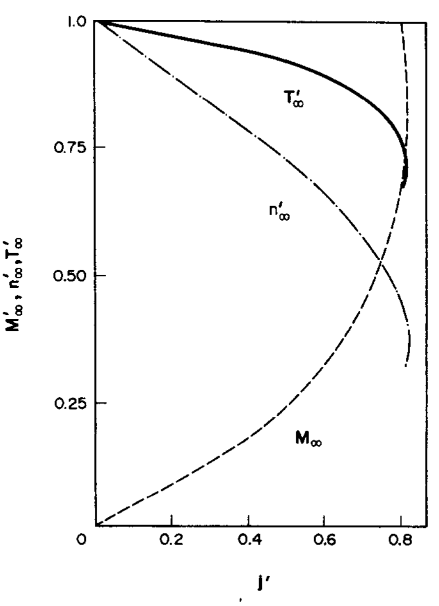


FIG. 2. Results of calculation of gasdynamic parameters of intensive evaporation.

For comparison the Table also contains the results of solution based on approximation of the distribution function given in [19].

$$f = n\left(\frac{m}{2\pi kT}\right)^{1/2}\cdot\left(\frac{m}{2\pi kT_\perp}\right) \times e^{-\left[\frac{m}{2kT}(\xi_x - u)^2 + \frac{m}{2kT_\perp}(\xi_y^2 + \xi_z^2)\right]}. \tag{2.14}$$

Figure 3 presents a graphical comparison of these results with the results of numerical solution given in [11] (the latter are limited by $j' \leqslant 0.7$ at which accuracy of calculations seems to be sufficient). One may note a good agreement of the results obtained by different methods. In the linear approximation, the approach under consideration gives the following expressions for dimensional quantities $P_\infty$, $P_s$, $T_\infty$, $T_s$, $j$ with $\beta = 1$

$$\frac{T_s - T_\infty}{T_s} = 0.445\hat{j}, \tag{2.15}$$

$$\hat{j} = \frac{1}{1 - 0.4}\cdot\frac{P_s - P_\infty}{P_s}\cdot\frac{1}{2(\pi)^{1/2}}, \tag{2.16}$$

where

$$\hat{j} = j/\rho_s C_s, \quad C_s = (2RT_s)^{1/2}.$$

Expressions (2.15)–(2.16) virtually coincide with the results obtained in the linear theory [3] on the basis of the eight-moment approximation [see expressions (1.3) with $q = 0$]. Similar calculations on the basis of distribution function (2.14) lead to less accurate agreement with the linear theory. However, it should be pointed out that application of this function enables one, on the basis of the moment

Table 1. Results of calculations of gasdynamic parameters for evaporation

|  | $T'_\infty$ |  | $n'_\infty$ |  | $M_\infty$ |  |
|---|---|---|---|---|---|---|
| $j'$ | I | II | I | II | I | II |
| 0.05 | 0.994 | 0.992 | 0.977 | 0.976 | 0.015 | 0.016 |
| 0.10 | 0.986 | 0.983 | 0.948 | 0.948 | 0.035 | 0.033 |
| 0.15 | 0.980 | 0.975 | 0.927 | 0.925 | 0.050 | 0.050 |
| 0.20 | 0.972 | 0.966 | 0.899 | 0.898 | 0.070 | 0.070 |
| 0.25 | 0.964 | 0.956 | 0.873 | 0.871 | 0.091 | 0.090 |
| 0.30 | 0.956 | 0.947 | 0.847 | 0.845 | 0.111 | 0.112 |
| 0.35 | 0.946 | 0.935 | 0.816 | 0.815 | 0.137 | 0.137 |
| 0.40 | 0.936 | 0.924 | 0.786 | 0.785 | 0.164 | 0.163 |
| 0.45 | 0.926 | 0.913 | 0.758 | 0.757 | 0.186 | 0.191 |
| 0.50 | 0.914 | 0.900 | 0.725 | 0.724 | 0.222 | 0.224 |
| 0.55 | 0.899 | 0.885 | 0.686 | 0.690 | 0.263 | 0.261 |
| 0.60 | 0.884 | 0.868 | 0.650 | 0.652 | 0.305 | 0.304 |
| 0.65 | 0.869 | 0.853 | 0.615 | 0.612 | 0.347 | 0.352 |
| 0.70 | 0.844 | 0.826 | 0.562 | 0.568 | 0.420 | 0.416 |
| 0.75 | 0.814 | 0.796 | 0.506 | 0.512 | 0.510 | 0.505 |
| 0.80 | 0.765 | 0.751 | 0.426 | 0.449 | 0.664 | 0.638 |
| 0.82 | 0.703 | 0.721 | 0.345 | 0.411 | 0.883 | 0.724 |
| 0.84 |  | 0.652 |  | 0.341 |  | 0.942 |

Note: I, calculation with the use of the present authors' approximation; II, results obtained with the use of Crout's approximation [19].

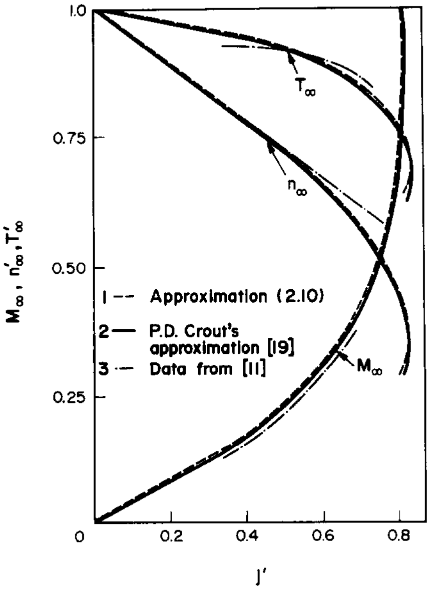


FIG. 3. Comparison of different approaches to solution of intensive evaporation problem. 1. Results of the present work. 2. Calculation with the use of Crout's approximation [19]. 3. Data of [11].

method solution of the Boltzmann equation, to obtain not only the values of the vapour gasdynamic parameters in the continuous flow region, but also their behaviour in the Knudsen layer. An example of such calculation of the number density $n$, carried out for the Maxwellian model of interaction of molecules, is presented in Fig. 4. Two alternative moment systems have been used for this solution and therefore two curves are presented for each value of $j'$ in this figure.

Additional information on the validity of the above approximate method of description may be obtained through resorting to the familiar Boltzmann $H$-theorem. For steady-state one-dimensional problems investigated in the present paper, this theorem is formulated as follows

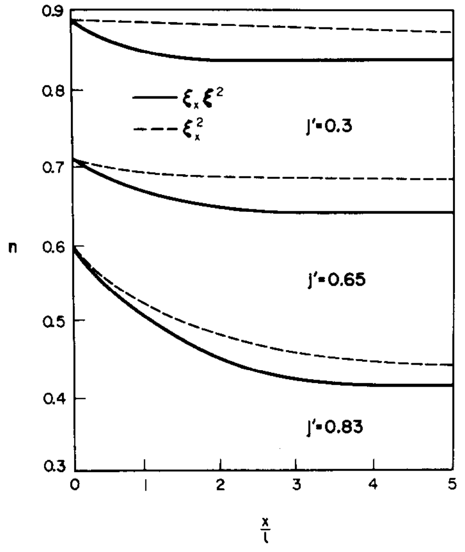


FIG. 4. Variation of vapour density along the coordinate $x$.

$$\frac{\partial \dot{H}_x}{\partial x} \leqslant 0, \tag{2.17}$$

where $\dot{H}_x$ is the specific flux of the $H$-function along the coordinate $x$. Equation (2.17) shows that $\Delta\dot{H}_x = \dot{H}_{0x} - \dot{H}_{\infty x} > 0$, where $\dot{H}_{0x}$, $\dot{H}_{\infty x}$ are the values of the specific flux of the $H$-function for $x = 0$ and $x \to \infty$, respectively. On the basis of the known distribution functions for $x = 0$ and $x \to \infty$ the values of $\Delta\dot{H}_x$ have been found.

Introducing the dimensionless specific flux of the $H$-function

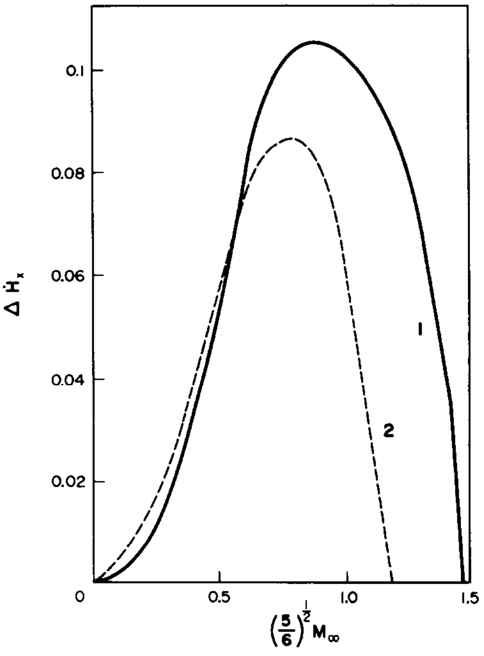
$$\dot{H}'_x = \frac{\dot{H}_x}{n_s(kT_s/2\pi m)^{1/2}}{}^{*}$$

we obtain for approximation (2.10)

$$\begin{aligned}\Delta\dot{H}_x = {} & \tfrac{3}{2}u_\infty n_\infty - u_\infty n_\infty \ln(n_\infty T_\infty^{-3/2}) \\ & -2\left[1 - Cn_\infty T_\infty^{1/2}\exp\left(-\frac{u_\infty^2}{4\pi T_\infty}\right)\right] \\ & -\tfrac{3}{4}Cn_\infty u_\infty\left\{1 - \operatorname{erf}\left[\frac{u_\infty}{2(\pi T_\infty)^{1/2}}\right]\right\} \\ & +(u_\infty n_\infty - 1)\ln(Cn_\infty T_\infty^{-3/2}). \end{aligned} \tag{2.18}$$

By analogy, for Crout's approximation (2.14):

$$\Delta\dot{H}_{x_{Cr}} = un[\ln(nT^{-1/2}T_\perp^{-1}) - \ln(n_\infty T_\infty^{-3/2})]. \tag{2.19}$$



--- On the basis of approximation (2.10)

—— On the basis of P.D. Crout's approximation [19]

FIG. 5. Application of the Boltzmann $H$-theorem to estimate the validity of different approximations. 1. Crout's approximation [19]. 2. Present authors' approximation (2.10).

In Fig. 5, the increment of the $H$-function flux, $\Delta\dot{H}_x$, is plotted vs the Mach number $M_\infty$. Variation of $\Delta\dot{H}_x$ corresponds to the physical concepts about the increase in the extent of the departure of vapour from the equilibrium state near the interface with increasing $M$ number of the vapour flow. The maximum on the curves probably corresponds to the regime of the limiting intensity of evaporation which restricts the region of virtually occurring flows. This regime, as is seen from Fig. 5, corresponds to the number $M_\infty \approx 1$, which yields $j' \simeq 0.82$ (about 18% of the evaporated molecules are again trapped by the surface). This value as well as $T_\infty \simeq 0.70T_s$ in the limiting evaporation regime is found to be in good agreement with the results of the numerical solution given in [12].

On the basis of the results obtained (Table 1), using the Clapeyron–Clausius equation

$$\frac{\mathrm{d}P}{\mathrm{d}T} = \frac{r}{T(v''-v')} \tag{2.20}$$

and the Trouton approximate rule

$$r/RT \simeq 10, \tag{2.21}$$

where $r$ is the specific heat of evaporation, $v''$ and $v'$ are specific volumes of vapour and liquid (solid), respectively, at temperature $T$, we may show that the vapour formed as a result of evaporation turns out to be supersaturated. Really, assuming that $v'' \gg v'$ and $v = RT/P$, we shall obtain the following solution of (2.20)

$$P = C\mathrm{e}^{-r/RT}, \tag{2.22}$$

where $C$ is the constant of integration.

Hence,

$$P_s(T_\infty) = C\mathrm{e}^{-r/RT_\infty}, \tag{2.23}$$

where $P_s(T_\infty)$ is the saturation vapour pressure corresponding to temperature $T_\infty$.

From (2.22) and (2.23) it is obvious that

$$\frac{P_\infty}{P_s(T_\infty)} = \frac{P_\infty}{P_s}\cdot\mathrm{e}^{\frac{r}{RT_s}\left(\frac{T_s}{T_\infty}-1\right)}. \tag{2.24}$$

Accounting for equation (2.21) and definitions of $T'_\infty$ and $n'_\infty$ from (2.12), we may rewrite relation (2.24) as

$$\frac{P_\infty}{P_s(T_\infty)} \simeq T'_\infty n'_\infty \mathrm{e}^{10\left(\frac{1}{T'_\infty}-1\right)}. \tag{2.25}$$

As mentioned above, in the problem of evaporation the knowledge of one parameter of vapour flow in the gasdynamic region ($T'_\infty$ in particular) is sufficient for all remainder characteristics of the flow to be determined, $j$ including. Thus, noting (2.25), a definite degree of vapour supersaturation $[P_\infty/P_s(T_\infty)]$ will correspond to each value of $j$. Figure 6 gives a plot of this value vs the mass flux density $j$. One can see that even before the limiting evaporation regime is reached, vapour departing from the boundary appears to be so supersaturated that actually there should occur spontaneous volume condensation (condensation shock) near the evaporation boundary. This conclusion agrees well with the qualitative estimates given in [14]. Thus, the implications of the linear theory are also confirmed in the region of intensive evaporation, but quantitatively all the effects being more pronounced.

For practical applications, we have developed, on the basis of the above results, the interpolation formulae for an evaporation process of arbitrary rate

$$\frac{T_s - T_\infty}{T_s} = \frac{(\pi)^{1/2}}{4}\cdot\frac{u_\infty}{C_s}, \tag{2.26}$$

---

* Hereafter, the primes on the dimensionless quantities will be omitted.

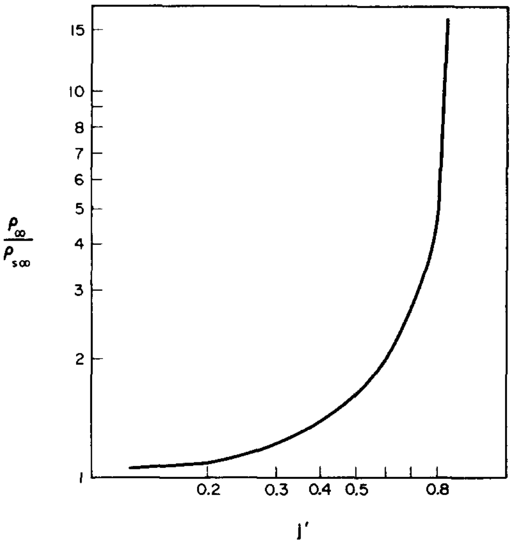


FIG. 6. The degree of supersaturation of vapour as a function of the intensity of mass flow.

$$\frac{T_s-T_\infty}{T_s} = 0.265\frac{\rho_s-\rho_\infty}{(\rho_s\rho_\infty)^{1/2}}, \tag{2.27}$$

$$j = 0.6C_s\left(\frac{\rho_\infty}{\rho_s}\right)^{1/2}(\rho_s-\rho_\infty), \tag{2.28}$$

[$C_s = (2RT_s)^{1/2}$, $\rho = m\cdot n$ is the vapour density].

The relations are written for $\beta = 1$. For arbitrary values of $\beta$, recalculation is made by the method suggested in [11] through substitution of $\rho_0$ for $\rho_s$ in relations (2.27) and (2.28)

$$\rho_0 = \left(1-2(\pi)^{1/2}\frac{j}{\rho_sC_s}\cdot\frac{1-\beta}{\beta}\right)\rho_s. \tag{2.29}$$

Relations (2.26)–(2.28) may be utilized in engineering applications. They are restricted to the limiting evaporation regime* which occurs at

$$j_{\lim} \simeq (0.80 \div 0.85)\frac{\rho_sC_s}{2(\pi)^{1/2}} \tag{2.30}$$

for $\beta = 1$. For $\beta < 1$, recalculation of (2.30) with the use of (2.29) is valid.

## III. INTENSIVE CONDENSATION

Analysis of intensive condensation will be performed following the technique applied in the previous section for treating evaporation. As before, we consider a steady-state one-dimensional problem of vapour flow towards the interface at which transition to a solid or liquid state takes place.

In accordance with the method adopted we shall first obtain the solution for the gasdynamic region of vapour flow and then analyze the processes in the Knudsen layer.

### (A) *Gasdynamic flow region*

The energy equation is now reduced to

$$\bar{h}+\frac{d\bar{h}}{dZ} = \bar{h}_\infty, \tag{3.1}$$

since $j < 0$ (condensation).

The solution of this equation satisfying the condition $(d\bar{h}/dZ)_{Z\to\infty} \to 0$ is

$$\bar{h}-\bar{h}_\infty = (\bar{h}_0-\bar{h}_\infty)e^{-Z}, \tag{3.2}$$

where $\bar{h}_0$ is the value of the function $\bar{h}$ at $Z = 0$. Further, express $\bar{h}/\bar{h}_\infty$ from (3.2) and substitute the result into the equation of motion (2.7). The resulting equation is

$$\frac{K-1}{K}H_\infty\left(1-\frac{\Delta\bar{h}}{\bar{h}_\infty}e^{-Z}-V\right) + \frac{K+1}{K}(V^2-V)+\frac{dV^2}{dZ} = 0, \tag{3.3}$$

where $\Delta\bar{h} = \bar{h}_\infty-\bar{h}_0$.

Having assumed that at a great distance from the interface the vapour flow is of the Euler type, we shall go over to the following condition that solution of equation (3.3) must satisfy

$$\left.\frac{dV}{dZ}\right|_{Z\to\infty} \to 0, \quad V|_{Z\to\infty} = 1. \tag{3.4}$$

Solution of (3.3)–(3.4) will be sought in the series form

$$V = 1+\sum_{n=1}^{\infty} a_ne^{-nZ}, \tag{3.5}$$

where $a_n$ are unknown series coefficients.

Substituting (3.5) into (3.3) and equating the coefficients of the same powers, $e^{-Z}$, we shall obtain expressions for $a_1.\ a_2\ldots a_n$

$$a_1 = -\frac{H_\infty}{1+H_\infty}\cdot\frac{\Delta\bar{h}}{\bar{h}_\infty}, \quad a_a = -\frac{a_1^2}{H_\infty+\dfrac{3K-1}{K-1}},$$

$$a_3 = -2a_1a_2\frac{\dfrac{2K-1}{K-1}}{\dfrac{5K-1}{K-1}+H_\infty}. \tag{3.6}$$

Relations (3.6) allow us to derive a recurrent formula for a general term of the series

$$a_n = \frac{\dfrac{(n-1)K-1}{K-1}}{\dfrac{(2n-1)K-1}{K-1}+H_\infty}\sum_{i=1}^{n-1}a_ia_{n-i}. \tag{3.7}$$

Series (3.5), in which the general term is expressed in accordance with (3.7), converges and, hence, expression (3.5) is a solution of (3.3). The proof of the convergence is based on comparison between the sum of the $(n+1)$ terms of the series,

$$\sum_{n=1}^{\infty} a_n,$$

with the value of $(a_1+a_2+\ldots a_n)^2$ and is considered in detail in [22]. The calculations involve, of course,

---

*More complete information on the magnitude of the limiting mass flux is to be found in works [20, 21] devoted to recondensation in a plane layer. The limiting regime in this system will exist at $Kn \to 0$ if one of the surfaces has $T \to 0$. For $Kn \simeq 0.1$, [20] gives $j \simeq 0.82$ and [21], $j \simeq 0.85$.

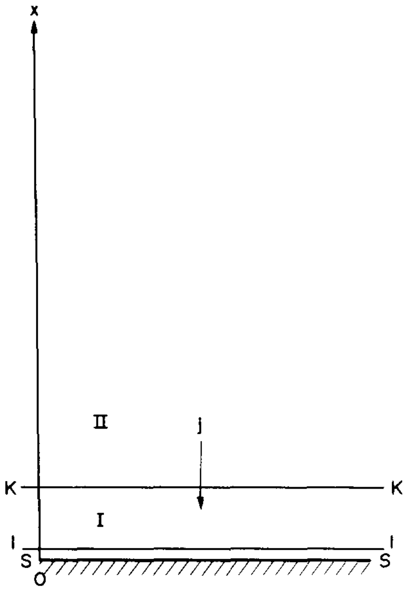


FIG. 7. Statement of the intensive condensation problem.

not an infinite series (3.5), but rather several of its partial sums. But in this case one has to know the magnitude of the resulting error. Let $\varepsilon$ be the sum of the terms in (3.5) from the second one to $n$, i.e.,

$$\varepsilon = a_2 + a_3 + \ldots + a_n. \tag{3.8}$$

It is shown in [22] that when $n \to \infty$

$$0 < \varepsilon < 1 - a_1 - (1 - 2a_1)^{1/2}. \tag{3.9}$$

This expression enables us to evaluate the maximum error appearing when series (3.5) is replaced by the sum $1 + a_1 e^{-Z}$. For example, for $Z = 0$ and $a_1 = 0.2$ it follows from (3.9) that this error is $\sim 3\%$, with $a_1 = 0.15$ it is $1.5\%$, etc. Replacing (3.5) by the sums $1 + a_1 e^{-Z} + a_2 e^{-2Z}$, $1 + a_1 e^{-Z} + a_2 e^{-2Z} + a_3 e^{-3Z}$, $1 + a_1 e^{-Z} + a_2 e^{-2Z} + a_3 e^{-3Z} + a_4 e^{-4Z}$ allows similar estimations to be made.

Thus solution of the energy and motion equations clearly shows that in condensation the Knudsen layer borders on the region where viscosity and thermal conductivity show up. At $x \to \infty$, this region (let us call it the Navier–Stokes region) transforms into the Euler flow.

### (B) *Knudsen layer*

Let us consider specific features of the flow in the Knudsen layer which are characteristic for the problem under study (Fig. 7). The matching of this layer with the Navier–Stokes region takes place at section $K-K$,* therefore it would be natural to represent the distribution function, $f_K$, at $K-K$ in the form of an ordinary thirteen-moment approximation. This, for the one-dimensional case considered, has the form

$$f_K = n_K \left( \frac{m}{2\pi k T_K} \right)^{3/2} e^{-\mathbf{v}^2} \left\{ 1 + \frac{1}{2} \frac{p_{11}}{P_K} (3v_1^2 - \mathbf{v}^2) + 2 \frac{q_1 v_1 (\frac{2}{5}\mathbf{v}^2 - 1)}{P_K (2RT_K)} \right\}, \tag{3.10}$$

where

$$P_K = n_K k T_K, \quad \mathbf{v} = \frac{\mathbf{V}}{(2RT_K)^{1/2}}, \quad \mathbf{V} = \xi - \mathbf{u}_K, \quad p_{ij} = P_{ij} - \delta_{ij} P.$$

When passing the Knudsen layer and colliding with the molecules that move from the condensation surface the function $f_K$ varies. Applying the same arguments as those used in the analysis of evaporation, we shall assume that at the condensation surface the distribution function of incident molecules is similar to function (3.10) for $\xi_x < 0$. Then, with regard for the diffusion nature of evaporation of molecules from the surface, the distribution function in the immediate vicinity of $S$–$S$ (Section I–I, Fig. 7) may be written as follows

$$\begin{aligned} f &= n_s \left( \frac{m}{2\pi k T_s} \right)^{3/2} e^{-\frac{\xi^2 m}{2kT_s}} & \xi_x &> 0, \\ f &= C f_K & \xi_x &< 0. \end{aligned} \tag{3.11}$$

The coefficient of proportionality $C$ is to be determined during solution. To determine the vapour parameters at the $K$–$K$ section, let us formulate the laws of mass, momentum and energy conservation for region I using (3.10) and (3.11). For this purpose, we shall write down the expressions for the mass, momentum and energy fluxes at Sections I–I $(j_1, j_{P_1}, j_{e_1})$ and $K$–$K$ $(j_K, j_{P_K}, j_{e_K})$. To simplify manipulations, we introduce the following dimensionless quantities† in addition to expressions (2.12)

$$\begin{gathered} T'_K = \frac{T_K}{T_s}, \quad n'_K = \frac{n_K}{n_s}, \quad u'_K = \frac{u_K}{(kT_s/2\pi m)^{1/2}}, \quad P'_K = \frac{P_K}{n_s k T_s}, \\ p'_{11} = \frac{p_{11}}{n_s k T_s}, \quad q'_1 = \frac{q_1}{4 n_s k T_s (k T_s / 2\pi m)^{1/2}}. \end{gathered} \tag{3.12}$$

---

*The boundary $K$–$K$ is an arbitrary one since matching occurs in a thin layer the thickness of which is of the order of the mean free path length.

†Hereafter, the primes on the dimensionless quantities will be omitted.

Then setting $j_1, j_{P_1}, j_{e_1}$ equal respectively to $j_K, j_{P_K}, j_{e_K}$ we shall obtain the system

$$
\begin{aligned}
&1 - Cn_K\left(T_K^{1/2}\phi - \frac{u_K}{2}\psi + \frac{\phi}{2}\frac{p_{11}}{P_K}T_K^{1/2} - \frac{2q_1u_K\phi}{5\pi P_K T_K^{1/2}}\right) = n_K u_K,\\
&\tfrac{1}{2} + Cn_K\left[\left(\frac{T_K}{2} + \frac{u_K^2}{4\pi} + \frac{p_{11}}{2P_K}T_K\right)\psi - \frac{u_K T_K^{1/2}}{2\pi}\phi - \frac{4q_1T_K^{1/2}}{5\pi P_K}\phi\right] = n_K T_K + \frac{n_K u_K^2}{2\pi} + \frac{p_{11}}{P_K}n_K T_K,
\end{aligned}
\tag{3.13}
$$

$$
\begin{aligned}
\tfrac{1}{2} - \frac{Cn_K}{2}\Bigg[ & T_K^{3/2}\left(1 + \frac{3}{4}\cdot\frac{p_{11}}{P_K}\right)\phi - \left(\tfrac{5}{8}T_K u_K + \frac{p_{11}}{4P_K}T_K u_K + \frac{u_K^3}{16\pi}\right)\psi + \frac{T_K^{1/2}u_K^2}{8\pi}\phi\\
&- \frac{u_K T_K^{1/2}\cdot q_1}{5\pi P_K}\phi - \frac{T_K q_1}{P_K}\psi\Bigg] = \tfrac{5}{8}n_K T_K u_K + \frac{n_K u_K^3}{16\pi} + \frac{n_K u_K T_K}{4}\frac{p_{11}}{P_K} + n_K T_K\frac{q_1}{P_K},
\end{aligned}
$$

where

$$
\phi = \exp\left(-\frac{u_K^2}{4\pi T_K}\right), \quad \psi = 1 + \operatorname{erf}\left[-\frac{u_K}{2(\pi T_K)^{1/2}}\right].
$$

Having formulated in a similar manner the conservation laws for region II, we obtain

$$
\begin{aligned}
&u_K n_K = u_\infty n_\infty, \qquad n_K T_K + \frac{n_K u_K^2}{2\pi} + n_K T_K\frac{p_{11}}{P_K} = n_\infty T_\infty + \frac{n_\infty u_\infty^2}{2\pi},\\
&\tfrac{5}{8}n_K T_K u_K + \frac{n_K u_K^3}{16\pi} + \frac{n_K u_K T_K}{4}\frac{p_{11}}{P_K} + n_K T_K\frac{q_1}{P_K} = \tfrac{5}{8}n_\infty T_\infty u_\infty + \frac{n_\infty u_\infty^3}{16\pi}.
\end{aligned}
\tag{3.14}
$$

Information on the gasdynamic flow is provided by (3.5). We shall consider this solution in the form

$$
V = 1 + a_1 e^{-Z}. \tag{3.15}
$$

With $Z = 0$, i.e., at section $K$–$K$ (3.15) yields

$$
\frac{u_K}{u_\infty} = 1 + a_1. \tag{3.16}
$$

Taking into consideration (3.6) and expressions for $\Delta\bar{h}$, $\bar{h}_\infty$, $H$ [see equations (2.4), (2.7), (3.3) above] equation (3.16) may be reduced to

$$
\frac{u_K}{u_\infty} = 1 - \frac{\dfrac{K}{K-1}(T_\infty - T_K) + \dfrac{u_\infty^2 - u_K^2}{4\pi}}{\dfrac{K}{K-1}T_\infty + \dfrac{u_\infty^2}{2\pi}}. \tag{3.17}
$$

The error resulting from replacement of (3.5) at $Z = 0$ by a simplified formula (3.16) may be estimated from (3.9). To increase the accuracy of the solution, it is necessary to expand the system and replace (3.16) by

$$
\frac{u_K}{u_\infty} = 1 + a_1 + a_2, \quad \frac{u_K}{u_\infty} = 1 + a_1 + a_2 + a_3, \ldots, \tag{3.18}
$$

where $a_1, a_2, \ldots, a_n$ are determined from (3.6). Then the number of equations increases from seven to eight, nine, ....

Solution of the conservation problem requires pre-assignment of two parameters. It follows from such reasons: in point of fact it is known [see Section II(A)] that solution of the evaporation problem requires the knowledge of one parameter on the outer boundary of the Knudsen layer. During condensation the viscosity and thermal conductivity become apparent but the existence of an additional correlation (3.5), (3.6) reduces the number of additionally pre-assigned parameters to one. Therefore, in the studied systems of the type (3.13), (3.14), (3.17) the number of unknown quantities exceeds the number of equations by 2, hence, the closure of these systems is achieved through two pre-assigned parameters. For the convenience of solution, we specified $u_K$ and $T_K$. Then the system reduces to a 6th-order equation, the solution of which yields all unknown quantities. Some of the results obtained in this way are given in Table 2. In the case of slow condensation, function (3.10) may be linearized to give

$$
f_K = f_s\left[1 + \dot{n} + 2i\xi'_x + \dot{\tau}(\xi'^2 - \tfrac{3}{2}) + \frac{4}{(\pi)^{1/2}}\bar{q}_1\xi'_x(\tfrac{2}{5}\xi'^2 - 1)\right], \tag{3.19}
$$

and (3.11) reduces to

$$
f = f_s\left[1 + \dot{n}_0 + 2i\xi'_x + \dot{\tau}(\xi'^2 - \tfrac{3}{2}) + \frac{4}{(\pi)^{1/2}}\bar{q}_1\xi'_x(\tfrac{2}{5}\xi'^2 - 1)\right] \quad \xi_x < 0, \tag{3.20}
$$

Table 2. Some results of calculations of gasdynamic parameters for condensation

| Regime No. parameter | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| $n_\infty$ | 1.7111 | 5.1365 | 3.2279 | 6.1327 | 1.3495 | 1.9127 | 1.4065 | 2.9223 | 5.4919 | 1.0597 |
| $T_\infty$ | 1.9162 | 2.5319 | 1.8546 | 1.6303 | 1.8535 | 1.7117 | 1.3145 | 1.1548 | 2.2082 | 1.0331 |
| $-u_\infty$ | 2.3873 | 4.6024 | 3.4625 | 3.8340 | 1.8488 | 2.2915 | 1.0951 | 1.9244 | 4.3959 | 0.1519 |
| $M_\infty$ | 0.5331 | 0.8939 | 0.7858 | 0.9280 | 0.4197 | 0.53974 | 0.2952 | 0.5535 | 0.9143 | 0.0462 |
| $h_\infty$ | 5.2413 | 8.0142 | 5.5896 | 5.2448 | 4.9046 | 4.7210 | 3.3806 | 3.1809 | 7.0573 | 2.5838 |
| $H_\infty$ | 11.5500 | 4.7521 | 5.8557 | 4.4815 | 18.0230 | 11.2930 | 35.4050 | 10.7890 | 4.5870 | 1406.0 |
| $n_K$ | 2.0414 | 4.2982 | 3.7255 | 5.8781 | 1.6663 | 2.1914 | 1.5403 | 2.8118 | 4.8284 | 1.0733 |
| $T_K$ | 1.6000 | 3.0000 | 1.6000 | 1.7000 | 1.5000 | 1.5000 | 1.2000 | 1.2000 | 2.5000 | 1.0200 |
| $-u_K$ | 2.0000 | 5.5000 | 3.0000 | 4.0000 | 1.5000 | 2.0000 | 1.0000 | 2.0000 | 5.0000 | 0.1500 |
| $p_{11}/P_K$ | 0.0801 | −0.2535 | 0.1423 | −0.0617 | 0.0581 | 0.0637 | 0.0129 | −0.0199 | −0.1877 | 0.00005 |
| $q_1/P_K$ | −0.2488 | 0.5188 | −0.3031 | 0.1018 | −0.2224 | −0.1861 | −0.0597 | 0.0469 | 0.3561 | −0.0012 |
| $-j$ | 4.0829 | 23.6400 | 11.1770 | 23.5130 | 2.4949 | 4.3828 | 1.5403 | 5.6236 | 24.1420 | 0.1610 |
| $j_p$ | 4.8282 | 30.3321 | 12.1490 | 24.3530 | 3.2358 | 4.8923 | 2.1173 | 5.0979 | 29.0260 | 1.0987 |
| $-j_e$ | 5.3532 | 47.3752 | 15.6220 | 30.8370 | 3.0599 | 5.1743 | 1.3218 | 4.4733 | 42.6040 | 0.1040 |
| $C$ | 0.9924 | 1.0044 | 0.9930 | 0.9989 | 0.9917 | 0.9920 | 0.9891 | 0.9923 | 1.0026 | 0.9954 |
| $a_1$ | −0.1622 | 0.1950 | −0.1336 | 0.0433 | −0.1886 | −0.1272 | −0.0868 | 0.0393 | 0.1374 | −0.0127 |

where

$$\bar{q}_1 = \frac{q_1}{4P_K(RT_K/2\pi)^{1/2}}, \quad \dot{n} = \frac{n_K - n_s}{n_s}, \quad \dot{\tau} = \frac{T_K - T_s}{T_s}, \quad \xi' = \frac{\xi}{(2RT_K)^{1/2}}, \quad \dot{p} = \dot{n} + \dot{\tau},$$

$$P_K = 1 + \dot{p}, \quad \dot{n}_0 = \frac{n_0 - n_s}{n_s}, \quad n_0 = Cn_K, \quad i = \frac{u_K}{(2RT_K)^{1/2}}, \quad f_s = n_s\left(\frac{m}{2\pi k T_s}\right)^{3/2} \mathrm{e}^{-\frac{\xi^2 m}{2kT_s}}.$$

In the linear approximation $u'_K \ll 1$, $u'_\infty \ll 1$, $\dot{n} \ll 1$, $\dot{\tau} \ll 1$, therefore $p_{11} = 0$, which makes it possible to obtain the following relationships from the conservation equations for the Knudsen layer

$$\hat{j} = -\frac{1}{0.6}(\dot{p} + 1.03\hat{q}_1)/2(\pi)^{1/2}, \tag{3.21}$$

$$\dot{\tau} = -0.444\hat{j} - 2.69\hat{q}_1, \tag{3.22}$$

where $\hat{j} = j/\rho_s C_s$, $\hat{q}_1 = \frac{2}{5} \cdot q_1/P_s C_s$.

Expressions (3.21) and (3.22) virtually coincide with the corresponding formulae obtained in [3] (at $\beta = 1$) [see expressions (1.3) above]. This indicates that the method considered adequately describes the linear processes either.

A characteristic feature of all the solutions derived for the condensation process is a good accuracy of the relationship

$$n_K T_K \cong n_\infty T_\infty,$$

and since $nT = P$, it means that $P_K \simeq P_\infty$. This allows us to conclude that the pressure in the process under investigation remains approximately constant. This conclusion, as is known, was arrived at in [11], when Kogan and Makashev analyzed the problem by a numerical technique. Then density and temperature along the axis $x$ may change in two ways: when approaching the interface either the density increases and temperature falls, or the density decreases and temperature increases, with $nT$ being naturally constant in either case.

On the basis of the calculation results and with the use of the description asymptotics in the low- and high-intensity regions we have derived an interpolation formula (with an error of $\leqslant 5\%$) for the density of mass flux at arbitrary intensity

$$j = 1.67\frac{P_s - P_\infty}{(2\pi R T_\infty)^{1/2}} \times \left\{1 + 0.515 \ln\left[\frac{P_\infty}{P_s}\left(\frac{T_s}{T_\infty}\right)^{1/2}\right]\right\}. \tag{3.23}$$

Equation (3.23) describes the case when $\beta = 1$. With $\beta \neq 1$, a recalculation is made according to the method of [11], viz., $\rho_s = P_s/RT_s$ in expression (3.23) is substituted for $\rho_0$ from (2.29). It should be noted that with the signs chosen the mass flux in condensation $j < 0$. The recalculations show that even with a slight decrease in $\beta$ from $\beta = 1$, the rate of condensation falls sharply which is attributed to the "shielding" effect of the flow of reflected molecules on the surface.

On the basis of relationship (3.23) we have undertaken an analysis of the experimental data of [23] on film condensation of mercury vapours over a flat vertical plate up to $M_\infty = 0.36$. (This seems to be the only one detailed investigation of the process of intensive condensation. Voluminous information on condensation of liquid metal vapours summarized in [24] refers to the conditions of validity of the linear theory of condensation.) The authors of [23] attempted, with the use of the linear theory relationships, to determine the condensation coefficient $\beta$ on the basis of their experimental results and obtained a "paradoxical" dependence of $\beta$ on the process intensity (mass flux). Figure 8 presents the results of treatment of the same experimental data with the use of relationships (3.23) and (2.29). One can easily see that account of the specificity of

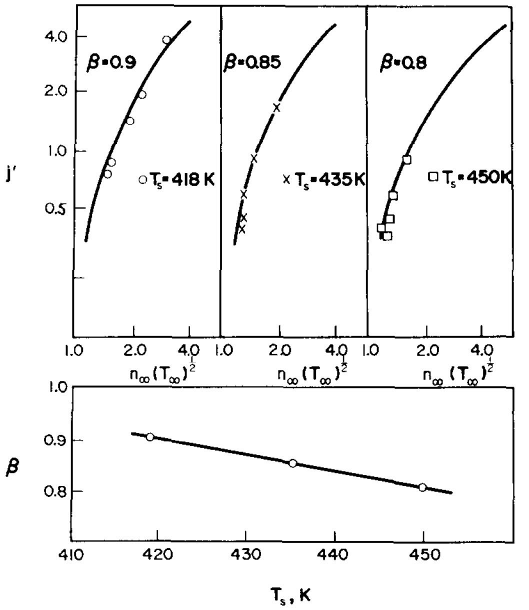


FIG. 8. Comparison of predicted results with the experimental data of Necmi and Rose [23].

intensive condensation corrects the result, i.e. the condensation coefficient becomes independent of the mass flux and somewhat decreases with increase in the surface temperature. The latter tendency seems to be quite reasonable. The above results served as an indirect check of the results of the present investigation.

The method considered permits the description of sufficiently intensive condensation processes when the linear theory is no longer valid. However, the proposed method is restricted by the numbers $|M_\infty| \leqslant 1$, i.e. by the region of subsonic condensation. This restriction is associated not only with formal reasons: at $|M_\infty| \gtrsim 1$, the region in which viscosity and thermal conductivity display themselves in the description of the outer region, turns out to be localized at a distance of several molecular free paths from the boundary of the Knudsen layer and division of the flow into regions I and II ceases to be valid. Of greater importance is the fact that the statement of the problems of one-dimensional condensation in a supersonic region has not as yet been physically clarified. Therefore, the results have been obtained for the flows characterized by the numbers $M_\infty \lesssim 1$.

## IV. BRIEF SUMMARY OF INVESTIGATION

The method suggested here for solution of problems of intensive evaporation and condensation has provided a closed description of these processes as a result of solution of the mass, momentum and energy conservation equations for the Knudsen layer and the gasdynamic vapour flow. This method, based on a choice of approximation for the distribution function in the Knudsen layer, is an approximate one. When dealing with specific engineering problems, where of interest is the relationship between the vapour parameters at a distance from the interface (in the gasdynamic flow region) and the thermodynamic characteristics of the interphase surface, the proposed approach provides a comprehensive information.

The results obtained for the intensive evaporation process are in good agreement with all available numerical solutions of this problem. The Boltzmann $H$-theorem, applied for the first time to the analysis of intensive processes, enabled us to determine the degree of reliability of various approximations of the distribution function and the value of the limiting mass flux in evaporation. Numerical calculations proved the vapour formed in evaporation to be supersaturated. The numerical value of the degree of supersaturation as a function of the process intensity has been obtained.

Using the present method the calculations of the intensive condensation have been made in a wide range of variation of the initial parameters (pressure and temperature of vapour at a distance from the interface). Comparison of these results with the data of Necmi and Rose [23] on intensive mercury condensation showed rather a good agreement.

On the basis of results of this investigation simple interpolation formulae are suggested for engineering calculations.

Solution of the basic equations of the continuum mechanics [Sections II(A) and III(A)] shows that the gasdynamic flow in evaporation differs from vapour behaviour in region II (Fig. 1) in condensation. In the first of these processes the flow is being realized at constant full enthalpy $h$ while in the second process $h$ varies over the coordinate $x$. Consequently, there is a physical asymmetry of these processes which is imperceptible at low velocities but is apparent at large rates. A similar statement seems to be first made in [25]. In addition, as it has already been mentioned, the problem of evaporation requires specification of one parameter for its solution while that of condensation requires two parameters for the purpose.

Except the above mentioned work of Necmi and Rose [23] the authors failed to find other reliable experimental data on intensive evaporation and condensation. Further experimental studies will make it possible to estimate the accuracy of the method suggested.

## REFERENCES

1. O. Knacke and I. N. Stransky, Mechanism of evaporation, *Prog. Met. Phys.* **6**, 181–235 (1956).
2. M. N. Kogan, *Dynamics of Rarefied Gas.* Nauka Press, Moscow (1967).
3. T. M. Muratova and D. A. Labuntsov, Kinetic analysis of vaporization and condensation processes, *Teplofiz. Vysok. Temper.* **7**, 959–967 (1969).
4. D. A. Labuntsov, An analysis of evaporation and condensation processes, *Teplofiz. Vysok. Temper.* **5**, 647–653 (1967).
5. J. R. Thomas, T. S. Chang and S. E. Siewert, Reverse temperature gradient in the kinetic theory of evaporation, *Phys. Rev. Lett.* **33**, 680–682 (1974).

6. J. W. Cipolla Jr., H. Lang and S. H. Loyalka, Kinetic theory of condensation and evaporation, *J. Chem. Phys.* **61**, 69–77 (1974).
7. Y. Sohe and Y. Onishi, Kinetic theory of evaporation and condensation, *J. Phys. Soc. Japan* **35**, 1773–1776 (1973).
8. R. Risch, Über die Kondensation von Quecksilber on einer vertikaler Wand, *Helv. Phys. Acta* **6**, 128–138 (1933).
9. R. W. Schrage, *A Theoretical Study of Interphase Mass Transfer*. Columbia University Press, New York (1953).
10. S. A. Zwick, Note on evaporation, *J. Appl. Phys.* **31**, 1735–1741 (1960).
11. M. N. Kogan and N. K. Makashev, Knudsen layer role in the theory of heterogeneous reactions and in the surface reaction flows, *Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza* **6**, 3–11 (1971).
12. S. I. Anisimov and A. Kh. Rakhmatulina, Expansion dynamics of vapour evaporating in vacuum, *Zh. Eksp. Teor. Fiz.* **64**, 869–876 (1973).
13. S. I. Anisimov, Evaporation of metal absorbing laser radiation, *Zh. Eksp. Teor. Fiz.* **54**, 339–342 (1968).
14. S. I. Anisimov, J. A. Imas, G. S. Romanov and U. V. Khodyko, *High-Energy Radiation Action on Metals*. Nauka Press, Moscow (1970).
15. Masahide Murakami and Koichi Oshima, Kinetic approach to the transient evaporation and condensation problem, *in Rarefied Gas Dynamics, Proceedings of the 9th International Symposium* II, pp. F.6-1–F.6-9 (1974).
16. G. A. Bird, Direct simulation and the Boltzmann equation, *Physics Fluids* **13**, 2676–2681 (1970).
17. A. M. Bishaev and V. A. Rykov, Study of the problem of one-dimensional sink on the basis of the kinetic equation, in *Computational Methods in the Rarefied Gas Dynamics*, edited by O. S. Ryzhov, pp. 9–21. Izd. Akad. Nauk SSSR, Moscow (1973).
18. D. A. Labuntsov and A. P. Kryukov, Processes of intensive evaporation, *Teploenergetika* **4**, 8–11 (1977).
19. P. D. Crout, An application of kinetic theory to the problems of evaporation and sublimation of monatomic gases, *J. Math. Phys.* **15**, 1–54 (1936).
20. F. G. Cheremisin, Rarefied gas flow between infinite plane-parallel emitting and absorbing surfaces, *Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza* **2**, 176–178 (1972).
21. J. Fischer, Distribution of pure vapour between two parallel plates under the influence of strong evaporation and condensation, *Physics Fluids* **19**, 1305–1311 (1976).
22. A. P. Kryukov, Solution of one-dimensional steady-state problems of viscous heat-conductive gas flow, *Trudy Mosk. Energ. Inst.* **347**, edited by D. A. Labuntsov, pp. 70–76, Moscow (1977).
23. S. Necmi and J. W. Rose, Film condensation of mercury, *Int. J. Heat Mass Transfer* **19**, 1245–1256 (1976).
24. M. N. Ivanovsky, V. P. Sorokin and V. I. Subbotin, *Evaporation and Condensation of Metals*. Atomizdat, Moscow (1976).
25. N. K. Makashev, Evaporation, condensation and heterogeneous chemical reactions with small Knudsen numbers, *Uchen. Zapiski TsAGI*, 49–62 (1974).

## ANALYSE DE L'EVAPORATION ET DE LA CONDENSATION INTENSIVES

**Résumé**—A partir des principes de la théorie de la cinétique moléculaire, on étudie les relations fondamentales qui gouvernent les mécanismes de l'évaporation et de la condensation intensives. Par une méthode approchée dévelopée par les auteurs, on obtient une description complète des mécanismes dans le cas des problèmes permanents et monodimensionnels. On présente les résultats des calculs, lesquels caractérisent les régularités fondamentales de ces mécanismes. Ils sont comparés aux résultats expérimentaux de Neomi et Rose [23] sur la condensation du mercure. A partir des résultats obtenus, on propose une formule simple d'interpolation. L'analyse présentée s'accorde bien avec les solutions numériques connues. Dans la région des mécanismes à faible intensité, les formules proposées s'accordent avec les relations connues de la théorie linéaire.

## UNTERSUCHUNG INTENSIVER VERDAMPFUNG UND KONDENSATION

**Zusammenfassung**—Unter Anwendung der Prinzipien der molekular-kinetischen Theorie werden die grundlegenden Zusammenhänge untersucht, die die Vorgänge intensiver Verdampfung und Kondensation bestimmen. Dabei werden stationäre eindimensionale Probleme berücksichtigt. Durch Anwendung einer Näherungsmethode, die von den Autoren entwickelt wurde, erhält man eine geschlossene Beschreibung der Vorgänge intensiver Verdampfung und Kondensation. Rechenergebnisse werden dargelegt, welche die Grundregeln obiger Prozesse beschreiben. Sie werden mit Versuchsdaten verglichen, die von Necmi und Rose [23] bei intensiver Kondensation von Quecksilber gewonnen wurden. Aufgrund der erlangten Ergebnisse werden einfache Interpolationsformeln vorgeschlagen. Die Resultate der Analyse stimmen gut mit den bekannten numerischen Lösungen für Vorgänge intensiver Verdampfung und Kondensation überein. Im Bereich von Vorgängen geringer Intensität gehen die vorgeschlagenen Formeln über in die bekannten Beziehungen nach der linearen Theorie.

# АНАЛИЗ ИНТЕНСИВНОГО ИСПАРЕНИЯ И КОНДЕНСАЦИИ

**Аннотация** — В работе изучаются закономерности интенсивных процессов испарения и конденсации с использованием аппарата молекулярно-кинетической теории. Рассматриваются одномерные стационарные задачи. С помощью разработанного приближенного метода получено замкнутое описание процессов интенсивного испарения и конденсации. Представлены расчётные данные, характеризующие закономерности изучаемых процессов. Проведено сравнение результатов расчёта с экспериментальными данными С. Незми и Дж. Роуза [23] по интенсивной конденсации ртути. На основе результатов исследования предложены простые интерполяционные формулы. Результаты анализа хорошо согласуются с известными численными решениями задач интенсивного испарения и конденсации. В области малоинтенсивных процессов предложенные формулы переходят в соотношения линейной теории, полученные ранее.